MODEL ANSWERS

AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname ______________________

Forename(s) ______________________

Candidate signature ______________________

AS MATHEMATICS

Paper 1

Wednesday 15 May 2019   Morning   Time allowed: 1 hour 30 minutes

Materials
- You must have the AQA Formulae for A-level Mathematics booklet.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
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TOTAL	

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Section A

Answer **all** questions in the spaces provided.

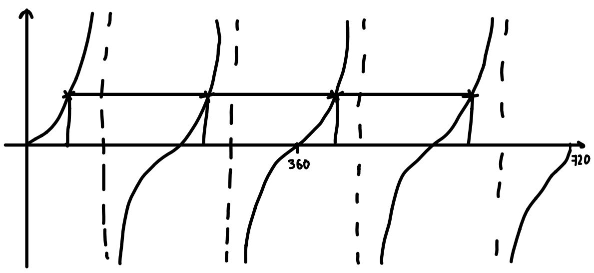
1 State the number of solutions to the equation $\tan 4\theta = 1$ for $0° < \theta < 180°$

Circle your answer.

[1 mark]

1 2 4 8

180 x 4 = 720



2 Dan believes that

for every positive integer n, at least one of $2^n - 1$ and $2^n + 1$ is prime.

Which value of n shown below is a counter example to Dan's belief?

Circle your answer.

[1 mark]

$n = 3$ $n = 4$ $n = 5$ $n = 6$

$2^6 - 1 = 63 = 7 \times 9$
$2^6 + 1 = 65 = 5 \times 13$

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3 It is given that $(x+1)$ and $(x-3)$ are two factors of $f(x)$, where

$$f(x) = px^3 - 3x^2 - 8x + q$$

3 (a) Find the values of p and q.

[3 marks]

Using the factor theorem:

$f(-1) = -p - 3 + 8 + q = 0 \Rightarrow p = 5 + q$ ①

$f(3) = 27p - 3(9) - 8(3) + q = 0 \Rightarrow 27p + q = 51$ ②

Substitute ① into ②: $27(5+q) + q = 51$

$135 + 27q + q = 51$

$28q = -84$

$q = -3$

Substitute $q=-3$ into ①: $p = 5 + (-3) = 2$, so $p = 2$, $q = -3$.

3 (b) Fully factorise $f(x)$.

[2 marks]

$f(x) = 2x^3 - 3x^2 - 8x - 3$

$$\begin{array}{r|l} & 2x^2 + 3x + 1 \\ x-3 & 2x^3 - 3x^2 - 8x - 3 \\ & 2x^3 - 6x^2 \\ & 0 \quad 3x^2 - 8x \\ & \quad 3x^2 - 9x \\ & \quad 0 \quad x - 3 \\ & \quad\quad x - 3 \\ & \quad\quad\quad 0 \end{array}$$

$f(x) = (x-3)(2x^2+3x+1)$

$f(x) = (x-3)(x+1)(2x+1)$

Turn over ▶

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4 Show that $\frac{\sqrt{6}}{\sqrt{3}-\sqrt{2}}$ can be expressed in the form $m\sqrt{n}+n\sqrt{m}$, where m and n are integers.

Fully justify your answer.

[4 marks]

$$\frac{\sqrt{6}}{\sqrt{3}-\sqrt{2}} = \frac{\sqrt{6}}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$$

$$= \frac{\sqrt{6}(\sqrt{3}+\sqrt{2})}{3-2}$$

$$= \sqrt{18} + \sqrt{12}$$

$$= \sqrt{9 \times 2} + \sqrt{4 \times 3}$$

$$= \sqrt{9}\sqrt{2} + \sqrt{4}\sqrt{3}$$

$$= 3\sqrt{2} + 2\sqrt{3}.$$

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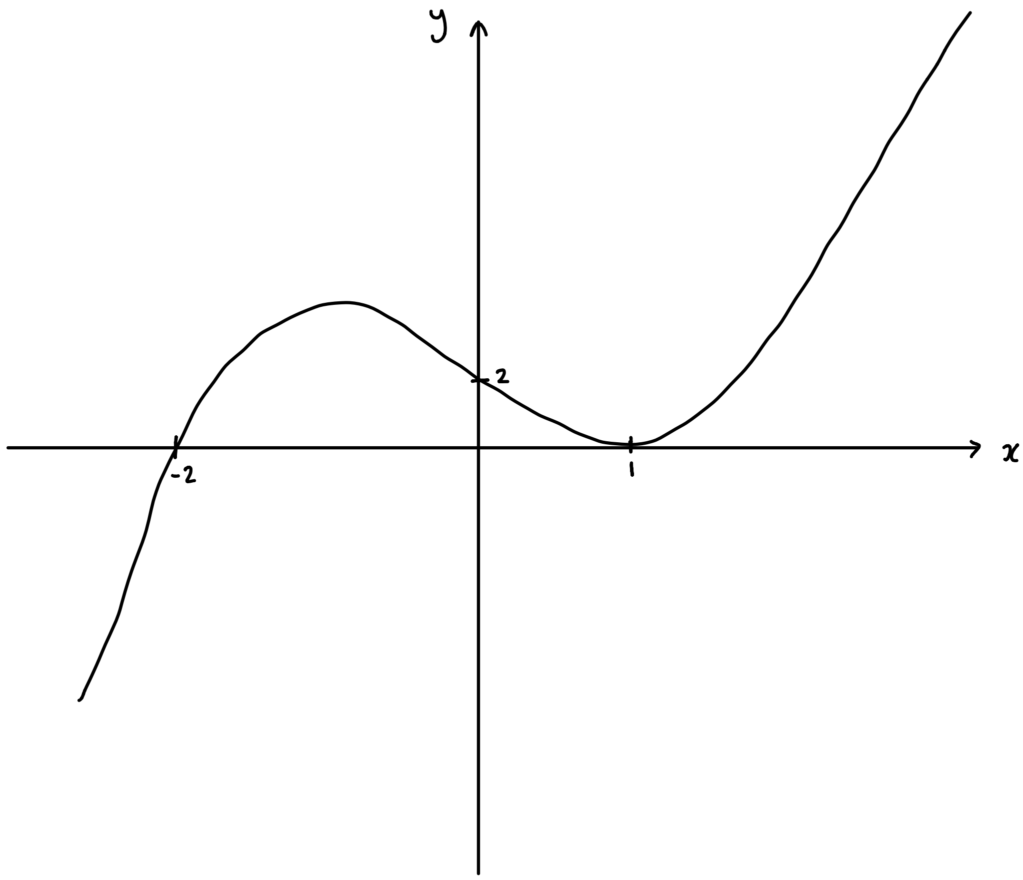
5 (a) Sketch the curve $y = g(x)$ where

$$g(x) = (x+2)(x-1)^2$$

[3 marks]



5 (b) Hence, solve $g(x) \leq 0$

[2 marks]

$g(x) \leq 0$ when the curve is on and below the x axis:

$x \leq -2$ or $x = 1$

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6 (a) (i) Show that $\cos\theta = \dfrac{1}{2}$ is one solution of the equation

$$6\sin^2\theta + 5\cos\theta = 7$$

[2 marks]

$6\sin^2\theta + 5\cos\theta = 7$

$6(1-\cos^2\theta) + 5\cos\theta = 7$

$6 - 6\cos^2\theta + 5\cos\theta = 7$

$6\cos^2\theta - 5\cos\theta + 1 = 0$

$(3\cos\theta - 1)(2\cos\theta - 1) = 0$

$\cos\theta = \frac{1}{3}$ or $\cos\theta = \frac{1}{2}$

So, $\cos\theta = \frac{1}{2}$ is indeed a solution.

6 (a) (ii) Find all the values of θ that solve the equation

$$6\sin^2\theta + 5\cos\theta = 7$$

for $0° \leq \theta \leq 360°$

Give your answers to the nearest degree.

[2 marks]

$\cos\theta = \frac{1}{2} \Rightarrow \theta = 60°, 300°$

$\cos\theta = \frac{1}{3} \Rightarrow \theta = 71°, 289°$

So solutions are : $\theta = 60°, 71°, 289°, 300°$

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6 (b) Hence, find all the solutions of the equation

$$6\sin^2 2\theta + 5\cos 2\theta = 7$$

for $0° \leq \theta \leq 360°$

Give your answers to the nearest degree.

[2 marks]

$\cos 2\theta = \frac{1}{2} \Rightarrow 2\theta = 60°, 300°, 420°, 660°$

$\Rightarrow \theta = 30°, 150°, 210°, 330°$ (to nearest degree)

$\cos 2\theta = \frac{1}{3} \Rightarrow 2\theta = 70°, 290°, 431°, 649°$

$\Rightarrow \theta = 35°, 145°, 215°, 325°$ (to nearest degree)

So solutions are : $\theta = 30°, 35°, 150°, 145°, 210°, 215°, 325°, 330°$

Turn over for the next question

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7 Given that $y \in \mathbb{R}$, prove that

$$(2+3y)^4 + (2-3y)^4 \geq 32$$

Fully justify your answer.

[6 marks]

LHS: $(2+3y)^4 + (2-3y)^4$

$= \binom{4}{0}2^4 + \binom{4}{1}2^3(3y) + \binom{4}{2}2^2(3y)^2 + \binom{4}{3}2(3y)^3 + \binom{4}{4}(3y)^4$

$+ \binom{4}{0}2^4 + \binom{4}{1}2^3(-3y) + \binom{4}{2}2^2(-3y)^2 + \binom{4}{3}2(-3y)^3 + \binom{4}{4}(-3y)^4$

$= 16 + 4(8 \times 3y) + 6(4 \times 9y^2) + 4(2 \times 27y^3) + 81y^4$

$+ 16 + 4(8 \times -3y) + 6(4 \times 9y^2) + 4(2 \times -27y^3) + 81y^4$

$= 16 + \cancel{96y} + 216y^2 + \cancel{216y^3} + 81y^4 + 16 - \cancel{96y} + 216y^2 - \cancel{216y^3} + 81y^4$

$= 32 + 432y^2 + 162y^4$

$y^2 \geq 0$ and $y^4 \geq 0$ for all y so:

$(2+3y)^4 + (2-3y)^4 = 32 + 432y^2 + 162y^4 \geq 32$

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8 Prove that the curve with equation

$$y = 2x^5 + 5x^4 + 10x^3 - 8$$

has **only one** stationary point, stating its coordinates.

[6 marks]

$\frac{dy}{dx} = 10x^4 + 20x^3 + 30x^2$

Stationary point occurs when $\frac{dy}{dx} = 0$:

$10x^4 + 20x^3 + 30x^2 = 0$

$x^4 + 2x^3 + 3x^2 = 0$

$x^2(x^2 + 2x + 3) = 0$

$x = 0$ or $x^2 + 2x + 3 = 0$

For $x^2 + 2x + 3$, discriminant is $b^2 - 4ac = 4 - 4(3) = -8 < 0$.

Since the discriminant is negative, it has no real solutions.

So only one stationary point at $x = 0$.

When $x = 0$, $y = -8$ so stationary point occurs at $(0, -8)$.

Turn over for the next question

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9 A curve cuts the x-axis at (2, 0) and has gradient function

$$\frac{dy}{dx} = \frac{24}{x^3}$$

9 (a) Find the equation of the curve.

[4 marks]

$\frac{dy}{dx} = 24x^{-3}$

$y = \int 24x^{-3}\,dx$

$y = -12x^{-2} + c$

$0 = -12(2)^{-2} + c$

$c = \frac{12}{4} = 3$

So, $y = -12x^{-2} + 3$

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9 (b) Show that the perpendicular bisector of the line joining $A(-2, 8)$ to $B(-6, -4)$ is the normal to the curve at $(2, 0)$

[6 marks]

Midpoint of AB: $\left(\frac{-2+(-6)}{2}, \frac{8+(-4)}{2}\right) = (-4, 2)$

Gradient of AB: $\frac{8-(-4)}{-2-(-6)} = \frac{12}{4} = 3$

So gradient of perpendicular bisector is $-\frac{1}{3}$

Equation of the perpendicular bisector: $y - 2 = -\frac{1}{3}(x - (-4))$

$y = -\frac{1}{3}x - \frac{4}{3} + 2$

$y = -\frac{1}{3}x + \frac{2}{3}$

When $x = 2$, $y = -\frac{1}{3}(2) + \frac{2}{3} = 0$ so the bisector passes through $(2, 0)$

Gradient of curve at $(2,0)$: $\frac{dy}{dx} = \frac{24}{2^3} = 3$

Gradient of the normal to the curve will be $-\frac{1}{3}$ so the bisector is the normal.

Turn over ▶

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10 On 18 March 2019 there were 12 hours of daylight in Inverness.

On 16 June 2019, 90 days later, there will be 18 hours of daylight in Inverness.

Jude decides to model the number of hours of daylight in Inverness, N, by the formula

$$N = A + B \sin t°$$

where t is the number of days after 18 March 2019.

10 (a) (i) State the value that Jude should use for A.

[1 mark]

12

10 (a) (ii) State the value that Jude should use for B.

[1 mark]

$18 = 12 + B\sin 90 \Rightarrow B\sin 90 = 6$

$\Rightarrow B = 6$

10 (a) (iii) Using Jude's model, calculate the number of hours of daylight in Inverness on 15 May 2019, 58 days after 18 March 2019.

[1 mark]

$12 + 6\sin(58) = 17.0882...$

$= 17.1$ (3.s.f)

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10 (a) (iv) Using Jude's model, find how many days during 2019 will have at least 17.4 hours of daylight in Inverness.

[4 marks]

$12 + 6\sin t = 17.4$

$6\sin t = 5.4$

$\sin t = 0.9$

$t = 64 \quad , \quad t = 116$

So days with at least 17.4 hours of sunlight is $116 - 64 = 52$ days.

10 (a) (v) Explain why Jude's model will become inaccurate for 2020 and future years.

[1 mark]

Jude's model repeats after 360 days but a year has 365 days.

10 (b) Anisa decides to model the number of hours of daylight in Inverness with the formula

$$N = A + B\sin\left(\frac{360}{365}t\right)^{\circ}$$

Explain why Anisa's model is better than Jude's model.

[1 mark]

The fraction in the sine function means Anisa's model will repeat after 365 days.

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Section B

Answer **all** questions in the spaces provided.

11 A ball moves in a straight line and passes through two fixed points, *A* and *B*, which are 0.5 m apart.

The ball is moving with a constant acceleration of $0.39\,\text{m s}^{-2}$ in the direction *AB*.

The speed of the ball at *A* is $1.9\,\text{m s}^{-1}$

Find the speed of the ball at *B*.

Circle your answer.

[1 mark]

$2\,\text{m s}^{-1}$ $\quad$ $3.2\,\text{m s}^{-1}$ $\quad$ $3.8\,\text{m s}^{-1}$ $\quad$ $4\,\text{m s}^{-1}$

$s = 0.5$
$u = 1.9$
$v = ?$
$a = 0.39$
$t = -$

$v^2 = u^2 + 2as$

$v = \sqrt{1.9^2 + 2(0.39)(0.5)} = 2$

12 A particle *P*, of mass m kilograms, is attached to one end of a light inextensible string.

The other end of this string is held at a fixed position, *O*.

P hangs freely, in equilibrium, vertically below *O*.

Identify the statement below that correctly describes the tension, T newtons, in the string as m varies.

Tick (✓) **one** box.

[1 mark]

- [ ] T varies along the string, with its greatest value at *O*
- [ ] T varies along the string, with its greatest value at *P*
- [ ] $T = 0$ because the system is in equilibrium
- [x] T is directly proportional to m

Turn over for the next question

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13 A car, starting from rest, is driven along a horizontal track.

The velocity of the car, $v\,\text{m s}^{-1}$, at time t seconds, is modelled by the equation

$$v = 0.48t^2 - 0.024t^3 \text{ for } 0 \le t \le 15$$

13 (a) Find the distance the car travels during the first 10 seconds of its journey.

[3 marks]

$S = \int_0^{10} 0.48t^2 - 0.024t^3 \, dt$

$= \left[0.16t^3 - 0.006t^4\right]_0^{10} = 0.16(10)^3 - 0.006(10)^4 = 100\text{ m}$

13 (b) Find the maximum speed of the car.

Give your answer to three significant figures.

[4 marks]

$\frac{dv}{dt} = 0.96t - 0.072t^2$

Maximum occurs when $\frac{dv}{dt} = 0$:

$0.96t - 0.072t^2 = 0$

$\frac{40}{3}t - t^2 = 0$

$t\left(\frac{40}{3} - t\right) = 0$

$t = 0$ or $t = \frac{40}{3}$

The car starts at rest so the maximum speed cannot be at $t=0$.

Maximum is at $t = \frac{40}{3}$. At $t = \frac{40}{3}$, $v = 0.48\left(\frac{40}{3}\right)^2 - 0.024\left(\frac{40}{3}\right)^3$

$v = 28.444\ldots$

$v = 28.4\text{ ms}^{-1}$.

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13 (c) Deduce the range of values of t for which the car is modelled as decelerating.

[2 marks]

$\frac{40}{3} < t \leq 15$

Turn over for the next question

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14 Two particles, *A* and *B*, lie at rest on a smooth horizontal plane.

A has position vector $\mathbf{r}_A = (13\mathbf{i} - 22\mathbf{j})$ metres
B has position vector $\mathbf{r}_B = (3\mathbf{i} + 2\mathbf{j})$ metres

14 (a) Calculate the distance between *A* and *B*.

[2 marks]

Distance between A and B $= \sqrt{(-22-2)^2 + (13-3)^2}$

$= \sqrt{(-24)^2 + 10^2}$

$= 26$

14 (b) Three forces, $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ are applied to particle *A*, where

$$\mathbf{F}_1 = (-2\mathbf{i} + 4\mathbf{j}) \text{ newtons}$$

$$\mathbf{F}_2 = (6\mathbf{i} - 10\mathbf{j}) \text{ newtons}$$

Given that *A* remains at rest, explain why $\mathbf{F}_3 = (-4\mathbf{i} + 6\mathbf{j})$ newtons

[1 mark]

At rest, the resultant force on A must be zero:

$F_1 + F_2 + F_3 = 0$

$(-2i + 4j) + (6i - 10j) + F_3 = 0$

$4i - 6j + F_3 = 0$

$F_3 = -4i + 6j$

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14 (c) A force of $(5\mathbf{i} - 12\mathbf{j})$ newtons, is applied to B, so that B moves, from rest, in a straight line towards A.

B has a mass of 0.8 kg

14 (c) (i) Show that the acceleration of B towards A is $16.25\,\text{m s}^{-2}$

[2 marks]

$F = 5i - 12j$

$|F| = \sqrt{5^2 + (-12)^2} = 13$

Newton's second law : $F = ma$

$13 = 0.8a$

$a = 16.25\,\text{ms}^{-2}$

14 (c) (ii) Hence, find the time taken for B to reach A.

Give your answer to two significant figures.

[2 marks]

$S = 26$ $\quad$ $S = ut + \frac{1}{2}at^2$

$u = 0$ $\quad$ $26 = 0(t) + \frac{1}{2}(16.25)t^2$

$v =$ $\quad$ $8.125t^2 = 26$

$a = 16.25$ $\quad$ $t^2 = 3.2$

$t = ?$ $\quad$ $t = 1.78885...$

$t = 1.8$ seconds

Turn over ▶

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15 A tractor and its driver have a combined mass of m kilograms.

The tractor is towing a trailer of mass $4m$ kilograms in a straight line along a horizontal road.

The tractor and trailer are connected by a horizontal tow bar, modelled as a light rigid rod.

A driving force of 11 080 N and a total resistance force of 160 N act on the tractor.

A total resistance force of 600 N acts on the trailer.

The tractor and the trailer have an acceleration of $0.8\,\text{m}\,\text{s}^{-2}$

15 (a) Find m.

[3 marks]

By Newton's second law: $F = ma$

$11080 - 160 - 600 = (4m + m)0.8$

$10320 = 5m(0.8)$

$4m = 10320$

$m = 2580$ kg

15 (b) Find the tension in the tow bar.

[2 marks]

For the trailer: $F = ma$

$T - 600 = (4 \times 2580)(0.8)$

$T - 600 = 8256$

$T = 8856$

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15 (c) At the instant the speed of the tractor reaches $18\,\text{km}\,\text{h}^{-1}$ the tow bar breaks.

The total resistance force acting on the trailer remains constant.

Starting from the instant the tow bar breaks, calculate the time taken until the speed of the trailer reduces to $9\,\text{km}\,\text{h}^{-1}$

[4 marks]

Using F=ma for the trailer after the towbar breaks:

$-600 = 10320a$

$a = -\frac{5}{86}\ \text{ms}^{-2}$

$18\,\text{kmh}^{-1} = \frac{18 \times 1000}{60 \times 60} = 5\text{ms}^{-1}$

$s = -$ $\quad$ $v = u + at$

$u = 5$ $\quad$ $2.5 = 5 - \frac{5}{86}t$

$v = 2.5$ $\quad$ $\frac{5}{86}t = 2.5$

$a = -\frac{5}{86}$ $\quad$ $t = 43$ seconds

$t = ?$

END OF QUESTIONS

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